PROBLEM 1)
Let $H'_{n,k}$ be the graph obtained by removing $k-1$ vertices. Some of these vertices may lie consecutively in the circle of $H_{n,k}$. But there can be no more than one such consecutive sequence of length $\geq k/2$. Otherwise, if there were two, then we would have removed $\geq k$ vertices instead of only $k-1$. Let x, y be vertices in $V(H'_{n,k})$. We can go from x to y as follows. We take a neighbor of x which lies on the circle, say we go clockwise. We go clockwise until we can't go on, and that is because we meet a sequence of vertices removed from $H_{n,k}$. If this sequence has length $< k/2$ then we can take an edge that 'jumps' over it (by construction of $H_{n,k}$). In this way we keep going. Two things can happen. In case 1, we meet y, in which case we are done. In case 2, we meet a sequence of length $\geq k/2$. Then we start again from x and go anticlockwise. By moving in the same way but anticlockwise, we will definitely meet y.

So $H_{n,k}$ is k-connected. To see that it has connectivity k, we remove a sequence $S_1 \subseteq V$ of $k/2$ vertices and again a sequence S_2 of $k/2$ vertices, but such that S_1 and S_2 don't form a sequence of length k together if $n \geq k+2$. Then $[S_1, S_2] =$ by construction of $H_{n,k}$, so H' is disconnected. If $n = k+1$ then we have $H_{n,k} = K_n$, which has connectivity $n-1 = k$.

PROBLEM 2)
Let $m \in \mathbb{N}_{\geq 0}$ be such that $\delta(G) = m + n/2$. Let x, y be vertices in $V(G)$ which are not neighbors. Vertices like this exist because otherwise $\delta(G)$ would be $n-1$. There are at least $2(m+n/2) = 2m+n$ edges going from x or y and there are exactly $n-2$ vertices where these edges can arrive. So there are at least $2m+n-(n-2) = 2\delta(G)-(n-2)$ vertices which are arrived by edges from both x and y. So this is also the number of vertices needed to remove such that there is no path between x and y anymore. Done.

Let n, d be such that $d \leq \frac{1}{2}n$. Consider $A_1 := K_{2d-(n-2)}$, $A_2 := K_{n-d-1}$ and $A_3 = A_2$. Join each vertex of A_1 to all vertices of A_2 and A_3. We call this constructed graph G. Then $|V(G)| = n$. We have $\delta(G) = (n-d-1)-1+(2d-(n-2)) = d$. We claim that by removing all the vertices of A_1 we get a disconnected graph, that is by removing $2d-(n-2)$ vertices. Note that $|A_1| = 2d-(n-2) = 2d-n+2 \leq 2$ so for this claim we only need $|A_2|$ (and $|A_3|$, which is equal) to be ≥ 1. So we are left with the case $|A_2| = n-d-1 = 0$. This is only if $G = K_n$. Then
$2d-(n-3) = 2(n-1)-(n-3) = n+1$, but now G is not $n+1$ connected.
PROBLEM 3)
Let xy be an arbitrary path. We can split this path into subpaths $x_1 ... x_2$, $x_2 ... x_3$, ..., $x_{n-1} ... x_n$ such that $x_i ... x_{i+1}$ is in the same block for all i. If each

block is k-edge connected then by removing $k-1$ edges of G we obtain say $G' = (V, E')$, of which the blocks are still connected. So for all i, we can change the subpath $x_i ... x_{i+1}$ by $y_i ... y_{i+1}$ such that these two are in the same block and such that $x_i = y_i$ and $x_{i+1} = y_{i+1}$. Thus G is still connected after removing $k-1$ edges.

Suppose now that we have $G = (V, E)$ and after removing $k-1$ edges we get $G' = (V, E')$ and a block B which breaks up into components $B_1, ..., B_k$. If G is still connected, there is a path P in G between two components B_i and B_j with some edges not in B, and these edges which are not in B exist, since otherwise the B_i were not components. But that is not possible, since then in G we get that $B \cup P$ would be a larger connected subgraph without cut vertices, in contradiction with the definition of a block.

PROBLEM 4)
Let's suppose it has k common vertices. Remove $k-1$ vertices arbitrarily. Then H and H' will still be connected and there is still a vertex x from the set of common vertices left. So if $v \in H$ and $w \in H'$, then we can make a path from v to w via x. So H is not a maximal k-connected subgraph. Contradiction.

PROBLEM 5)
Dit is nummer 4.1.26 van de volgende uitwerkingen:
http://home.ku.edu.tr/mudogan/public_html/Introduction%20to%20Graph%20Theory%202E%20-%20West.pdf